Stochastik SoSe09 - Hausaufgabe Blatt IV

David Pollehn - 2944852 - Bachelor Informatik

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Aufgabe 1

Aufgabe 2

a)

$$\Omega_X = \{0, 1\}$$
$$\Omega_Y = \{0, 1, 2\}$$

b)

$$P(X = 0 \wedge Y = 0) = \frac{1}{2^2}$$
$$P(X = 0 \wedge Y = 1) = \frac{2}{2^2}$$
$$P(X = 0 \wedge Y = 2) = 0$$
$$P(X = 1 \wedge Y = 0) = 0$$
$$P(X = 1 \wedge Y = 1) = 0$$
$$P(X = 1 \wedge Y = 2) = \frac{1}{2^2}$$

c)

$$P(X = 0) = \frac{3}{4}$$
$$P(X = 1) = \frac{1}{4}$$
$$E\{X\} = 1 \cdot P(X = 1) = \frac{1}{4}$$
$$Var\{X\} = E\{(X - E\{X\})^2\}$$
$$= \frac{3}{4} \cdot \frac{1}{16} + \frac{1}{4} \cdot \frac{9}{16}$$
$$= \frac{3}{16}$$
$$P(Y = 0) = P(Y = 2) = \frac{1}{4}$$
$$P(Y = 1) = \frac{2}{4}$$
$$E\{Y\} = \frac{2}{4} + \frac{2}{4} = 1$$
$$cov\{X, Y\} = E\{X, Y\} - E\{X\} \cdot E\{Y\}$$
$$= 2 \cdot \frac{1}{4} - \frac{1}{4} = \frac{1}{4}$$

Aufgabe 3

$$g_X(t) = \sum_{i=1}^{n} (\frac{1}{n} \cdot t^i)$$
$$g'_X(t) = \frac{1}{n} \cdot \sum_{i=1}^{n} (i \cdot t^{i-1})$$
$$E\{X\} = g'_X(1-) = \frac{n+1}{2}$$

$$g''_X(t) = \frac{1}{n} \cdot \sum_{i=1}^{n} (i \cdot (i-1) \cdot t^{i-2})$$
$$M_{(2)} = g''_X(1-) = \frac{1}{n} \cdot \sum_{i=1}^{n} (i \cdot (i-1)) = \frac{1}{n} \cdot \sum_{i=1}^{n-1} (i \cdot (i+1)) = \frac{1}{n} \cdot \left(\sum_{i=1}^{n-1} (i) + \sum_{i=1}^{n-1} (i^2) \right)$$
$$M_{(2)} = \frac{1}{n} \cdot \left(\frac{(n-1) \cdot n \cdot (2 \cdot (n-1) + 1)}{6} + \frac{n \cdot (n-1)}{2} \right) = \frac{n^2+1}{3}$$
$$Var\{X\} = M_{(2)} + E\{X\} \cdot (1 - E\{X\})$$
$$= \frac{n^2+1}{3} + \frac{n+1}{2} \cdot \frac{1-n}{2} = \frac{n^2+7}{12}$$

Aufgabe 4

$$Var\{X\} = \frac{2}{9}$$
$$E\{X\} = \frac{1}{3}$$
$$P(|X - \frac{1}{3} \cdot n| \leq \frac{n}{100}) \geq 0.95$$

$$\Rightarrow P(|X - \frac{1}{3} \cdot n| > \frac{n}{100}) < 0.05$$
$$\Rightarrow P(|X - \frac{1}{3} \cdot n| \geq \frac{n}{100}) < 0.05 + P(|X - \frac{1}{3} \cdot n| = \frac{n}{100})$$

Mit der Chebychev-Ungleichung:

$$\Rightarrow \frac{20000}{9 \cdot n} < \frac{5}{100} + 2 \cdot P(X = 0.31 \cdot n)$$
$$\Rightarrow n > 44445$$

Aufgabe 5